Multi-level Passive Order Reduction of Interconnect Networks

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***Abstract* — This paper presents an efficient algorithm for transient simulation of multi-port interconnect networks in the presence of non-linear terminations. Krylov-subspace order reduction techniques have been shown to provide a significant speed-up in the simulation of interconnect networks. These methods however are far from optimal, and the resulting macromodel contains many redundant poles. In this paper, a passive multi-level reduction technique is presented. The proposed method eliminates the redundant poles, thus resulting in significant CPU cost reduction.**

I. Introduction

Due to the rapidly increasing operating frequencies, the electrical length of interconnects is becoming a significant fraction of signal wavelength. Consequently, interconnect effects are becoming the dominant factors influencing signal integrity in high-speed systems [1], [2]. As a result, efficient and accurate transient simulation of interconnect and packaging effects is becoming an essential step in the design cycle. However, time-domain simulation requires discretization of the distributed interconnects [1], leading to large lumped RLC networks that may require prohibitively high CPU cost. To improve the CPU efficiency, model-order reduction techniques were proposed in the literature [3]–[7]. The objective of order reduction techniques is to generate a small system that matches the behavior of the original large system. This goal can be accomplished because, in general, only a fraction of the poles of the original system have a significant effect on the response over the frequency range of interest[1].

Recently, Krylov subspace-based methods have been introduced as an effective tool for model-order reduction [3]–[7]. However, a generally found difficulty with Krylov subspace methods is the issue of redundant poles [3], [7]. When the order of the approximation is increased to enhance its accuracy range, a large number of redundant poles are captured. This is illustrated in Fig. 1, which shows the region in which the poles are captured using Krylov subspace techniques, in contrast with the region near the imaginary axis where the dominant poles tend to concentrate. The difficulty arising due to the presence of redundant poles is multifold: 1) The number of states is too many to achieve a fast non-linear simulation, and 2) The poles, which are far away from the imaginary axis, can lead to smaller step sizes, resulting in slower transient simulation.

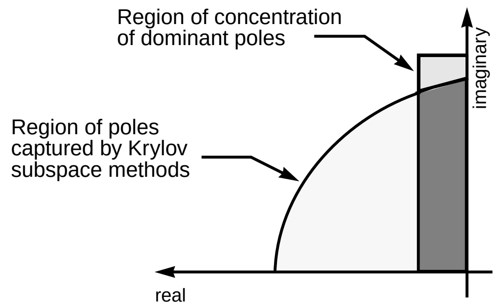

Fig. 1. Comparison of pole distribution

Some attempts have been made in the literature to address the problem of redundant poles [7]. These methods, based on the truncated balance realization (TBR) [8], have been successful in significantly reducing the number of states in the macromodel. However, the macromodels produced using TBR methods are not guaranteed to be passive. This is a major problem because non passive macromodels can lead to instability in the simulation when combined with nonlinear terminations [3], [4].

In this paper, a multi-level order reduction algorithm is proposed to address the problem of redundant poles, while guaranteeing the passivity of the resulting compact macromodel. Application of the proposed technique to interconnect circuits resulted in significantly smaller macromodels and lower computational cost compared to conventional Krylov subspace methods.

II. Problem Formulation

Consider a multiport interconnect network with nonlinear terminations (Fig. 2). The interconnect network consists of lumped and distributed components. A lumped representation of the linear subnetwork can be obtained by discretizing the distributed components through simple segmentation or through the use of more efficient techniques such as the Padé Model[9].

The Modified Nodal Analysis (MNA) equations of the resulting multiport linear lumped subnetwork can be expressed as

This work was supported in part by an IBM cooperative fellowship, and in part by Micronet, a Canadian Center of Excellence on Microelectronics Devices, Circuits, and Systems.

$$\mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u}$$
$$\mathbf{I} = \mathbf{B}^T\mathbf{x} \tag{1}$$

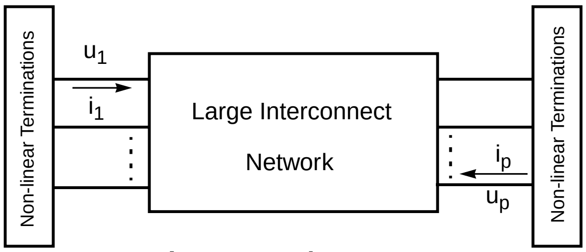

Fig. 2. Large multi-port network.

where:

1) $\mathbf{x} \in \Re^n$ is the vector of node voltage waveforms appended by independent voltage source currents, linear inductor currents, and port currents;
2) $\mathbf{C}, \mathbf{G} \in \Re^{n \times n}$ are constant matrices describing the lumped memory and memoryless elements of the network (**C** and **G** are constructed using the formulation suggested in [4] in order to ensure the passivity of the reduced order model);
3) **u** and **I** are vectors containing the port voltages and currents;
4) $\mathbf{B} = [b_{i,j} \in \{0, 1\}]$, where $i \in \{1, \ldots, n\}$, $j \in \{1, \ldots, p\}$ is a selector matrix that maps the port voltages into the node space $\Re^n$ of the network and p is the number of ports;

The objective of model reduction is to obtain a passive reduced order macromodel of the form

$$\tilde{\mathbf{G}}\tilde{\mathbf{x}} + \tilde{\mathbf{C}}\dot{\tilde{\mathbf{x}}} = \tilde{\mathbf{B}}\mathbf{u}$$
$$\mathbf{I} = \tilde{\mathbf{B}}^T\tilde{\mathbf{x}} \tag{2}$$

Macromodels produced using Krylov subspace techniques contain a large number of redundant poles[7]. The multi-level reduction technique presented in this paper eliminates the redundant poles while maintaining the passivity of the macromodel.

III. Overview Of The Algorithm

The reduction algorithm proposed in this paper consists of two main steps. First the Krylov subspace is obtained using the Arnoldi process[10], and a passive reduced order model is obtained through congruent transformation. Another level of reduction is then performed in order to remove the redundant poles while maintaining passivity.

A. First Level of Reduction

The first level of reduction is done by projecting the original system in (1) onto an orthonormal basis **K** of the Krylov subspace. The Block Arnoldi process with double orthogonalization[6] is used to obtain **K** which is defined as

$$colsp[\mathbf{K}] = colsp[\mathbf{R}, \mathbf{A}\mathbf{R}, \mathbf{A}^2\mathbf{R}, \ldots, \mathbf{A}^{k-1}\mathbf{R}], \tag{3}$$

where

$$\mathbf{A} = \mathbf{G}^{-1}\mathbf{C} \qquad \mathbf{R} = \mathbf{G}^{-1}\mathbf{B}. \tag{4}$$

The reduced system is then obtained through congruent transformation such that

$$\hat{\mathbf{G}}\hat{\mathbf{x}} + \hat{\mathbf{C}}\dot{\hat{\mathbf{x}}} = \hat{\mathbf{B}}\mathbf{u}$$
$$\mathbf{I} = \hat{\mathbf{B}}^T\hat{\mathbf{x}}, \tag{5}$$

where

$$\hat{\mathbf{G}} = \mathbf{K}^{\mathbf{T}}\mathbf{G}\mathbf{K} \qquad \hat{\mathbf{C}} = \mathbf{K}^{\mathbf{T}}\mathbf{C}\mathbf{K}$$
$$\hat{\mathbf{B}} = \mathbf{K}^{\mathbf{T}}\mathbf{B}. \tag{6}$$

The reduced order system in (5), while much smaller than the original system, still contains many redundant poles[3][7]. This severely reduces the efficiency of the transient simulation with nonlinear terminations. This problem is addressed in the following section, where a second level of reduction is used to remove the redundant poles while maintaining passivity.

B. Second Level of Reduction

In a second level of reduction, the dominant poles are selected based on their contribution to the response. The system in (5) is then further reduced by projection onto an orthonormal basis **Q** of the eigenspace of the dominant poles. The resulting system is passive and preserves only the dominant poles of the original network. The basis **Q** is defined as

$$colsp[\mathbf{Q}] = colsp[\mathbf{v}_1, \mathbf{v}_2 \ldots, \mathbf{v}_m], \tag{7}$$

where $\mathbf{v}_i$ are eigenvectors corresponding to the dominant eigenvalues of $\hat{\mathbf{A}} = \hat{\mathbf{G}}^{-1}\hat{\mathbf{C}}$. The matrix **Q** is obtained from the eigenvectors using an orthogonalization process such as the Gram-Schmidt or the Householder techniques[10]. It is to be noted that while the eigenvectors can be complex, **Q** is made real by splitting the real and imaginary parts of complex eigenvectors. The reduced order system is then obtained as

$$\tilde{\mathbf{G}}\tilde{\mathbf{x}} + \tilde{\mathbf{C}}\dot{\tilde{\mathbf{x}}} = \tilde{\mathbf{B}}\mathbf{u}$$
$$\mathbf{I} = \tilde{\mathbf{B}}^T\tilde{\mathbf{x}}, \tag{8}$$

where

$$\tilde{\mathbf{G}} = \mathbf{Q}^{\mathbf{T}}\hat{\mathbf{G}}\mathbf{Q} \qquad \tilde{\mathbf{C}} = \mathbf{Q}^{\mathbf{T}}\hat{\mathbf{C}}\mathbf{Q}$$
$$\tilde{\mathbf{B}} = \mathbf{Q}^{\mathbf{T}}\hat{\mathbf{B}} \tag{9}$$

The final reduced system in (8) is passive by construction since the orthonormal projection matrix **Q** is real[4].

IV. Proof of Preservation of Poles

Consider a matrix $\hat{\mathbf{A}} = \hat{\mathbf{G}}^{-1}\hat{\mathbf{C}} \in \Re^{k \times k}$ which has an eigenvalue λ_i and corresponding eigenvector $\mathbf{v}_i$ such that

$$\hat{\mathbf{A}}\mathbf{v}_{\mathbf{i}} = \lambda_{\mathbf{i}}\mathbf{v}_{\mathbf{i}}, \text{ or } \hat{\mathbf{C}}\mathbf{v}_i = \lambda_i\hat{\mathbf{G}}\mathbf{v}_i. \tag{10}$$

An orthonormal basis $\mathbf{Q} \in \Re^{k \times q}$ containing the eigenvector $\mathbf{v}_{\mathbf{i}}$ ($\mathbf{v}_{\mathbf{i}} \in \mathbf{Q}$) can be constructed such that

$$\mathbf{v}_{\mathbf{i}} = \mathbf{Q}\tilde{\mathbf{v}}_{\mathbf{i}}, \tag{11}$$

where $\tilde{\mathbf{v}}_{\mathbf{i}} \in \Re^{q}$. Substituting (11) into (10), and premultiplying by $\mathbf{Q}^{T}$, we get

$$\tilde{\mathbf{C}}\tilde{\mathbf{v}}_i = \lambda_i\tilde{\mathbf{G}}\tilde{\mathbf{v}}_i \text{ or } \tilde{\mathbf{A}}\tilde{\mathbf{v}}_{\mathbf{i}} = \lambda_{\mathbf{i}}\tilde{\mathbf{v}}_{\mathbf{i}} \tag{12}$$

where

$$\tilde{\mathbf{A}} = \tilde{\mathbf{G}}^{-1}\tilde{\mathbf{C}}$$
$$\tilde{\mathbf{G}} = \mathbf{Q}^{\mathbf{T}}\hat{\mathbf{G}}\mathbf{Q} \qquad \tilde{\mathbf{C}} = \mathbf{Q}^{\mathbf{T}}\hat{\mathbf{C}}\mathbf{Q} \tag{13}$$

The reduced matrix $\tilde{\mathbf{A}} \in \Re^{q \times q}$ has therefore λ_i as an eigenvalue, with $\tilde{\mathbf{v}}_{\mathbf{i}}$ as a corresponding eigenvector. Therefore, an orthonormal projection onto the eigenspace of some selected eigenvectors preserves the eigenvalues corresponding to these eigenvectors. The fact that this can be done through congruent transformation is critical for the preservation of passivity.

V. Proof of Preservation of Passivity

By substituting the values of $\hat{\mathbf{G}}$, $\hat{\mathbf{C}}$, and $\hat{\mathbf{B}}$ defined in (6) into equation (9) the definition of the reduced system can be written as

$$\tilde{\mathbf{G}} = \mathbf{Q}^{\mathbf{T}}\mathbf{K}^{\mathbf{T}}\mathbf{G}\mathbf{K}\mathbf{Q} \qquad \tilde{\mathbf{C}} = \mathbf{Q}^{\mathbf{T}}\mathbf{K}^{\mathbf{T}}\mathbf{C}\mathbf{K}\mathbf{Q}$$
$$\tilde{\mathbf{B}} = \mathbf{Q}^{\mathbf{T}}\mathbf{K}^{\mathbf{T}}\mathbf{B} \tag{14}$$

where **K** and **Q** are the orthonormal projection matrices used in the first and second levels of reduction. By noting that $\mathbf{S} = \mathbf{K}\mathbf{Q}$ is also an orthonormal basis, the reduced system can be written as a direct projection on the subspace **S**

$$\tilde{\mathbf{G}} = \mathbf{S}^{\mathbf{T}}\mathbf{G}\mathbf{S} \qquad \tilde{\mathbf{C}} = \mathbf{S}^{T}\mathbf{C}\mathbf{S}$$
$$\tilde{\mathbf{B}} = \mathbf{S}^{\mathbf{T}}\mathbf{B} \tag{15}$$

The reduced order system can therefore be obtained by congruent transformation using a real orthonormal basis **S**. Due to the structure of the G and C matrices such reduced order system is passive[4].

VI. Numerical Example

In order to demonstrate the accuracy and efficiency of the proposed order reduction technique, a large interconnect network consisting of 1448 nodes was considered. The proposed algorithm was used to obtain a reduced order macromodel of order 50 that matches the response of the original system. In Fig. 4 and Fig. 5, the Y-parameters of the original and reduced systems are shown to be matching with no noticeable difference. In order to obtain a similar accuracy using Krylov subspace techniques a system of order 150 is required. The proposed method therefore was able to eliminate 100 redundant states thus reducing the order of the system by 67% compared to conventional Krylov methods.

To illustrate the computational efficiency of the reduced-order system, the interconnect network was connected to nonlinear terminations as shown in Fig. 3. The simulation results for the original and reduced systems at the output node (V_{out}) and at node n_2 are compared in Fig. 6 and Fig. 7. The transient simulation of the reduced system was 77 times faster than that of the original system with almost no loss of accuracy.

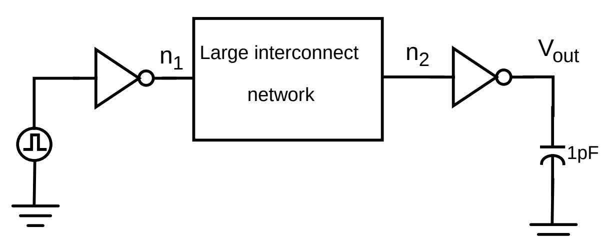

Fig. 3. Large interconnect network with nonlinear termination.

VII. Conclusion

In this paper, a multi-level order reduction technique was presented. The new method introduces a second level of reduction that eliminates the redundant poles obtained by Krylov subspace methods. Furthermore, the macromodel produced by the proposed algorithm is passive, and therefore stable in nonlinear transient simulations.

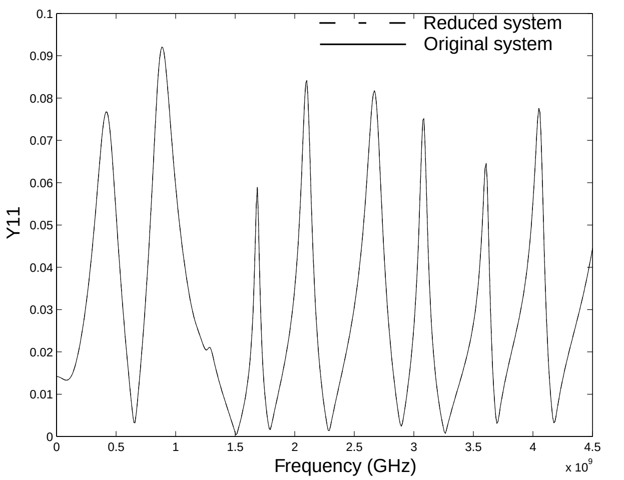

Fig. 4. Comparison of Y11 of the original and reduced systems

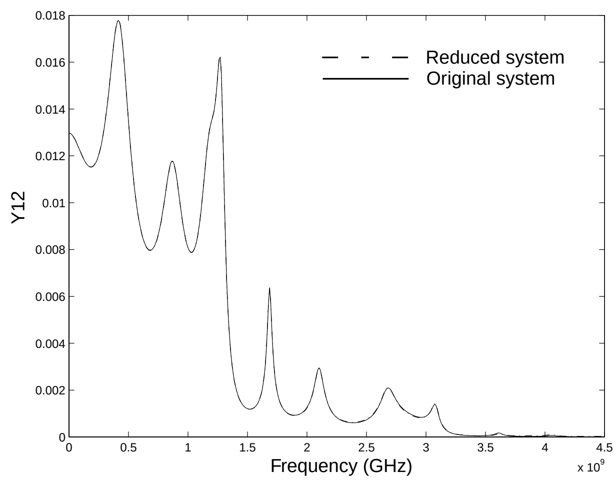

Fig. 5. Comparison of Y12 of the original and reduced systems

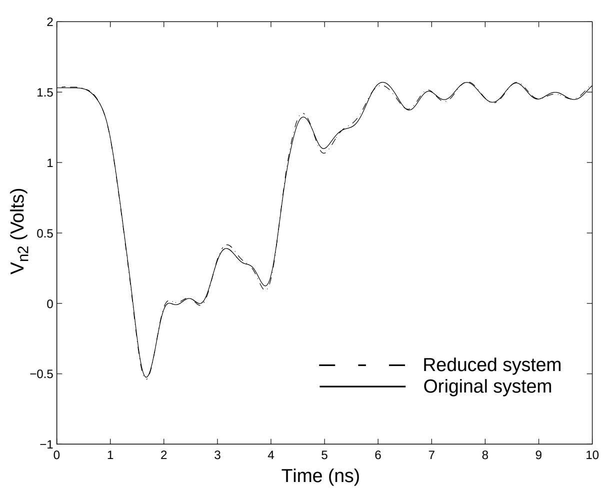

Fig. 6. Transient response at node n_2

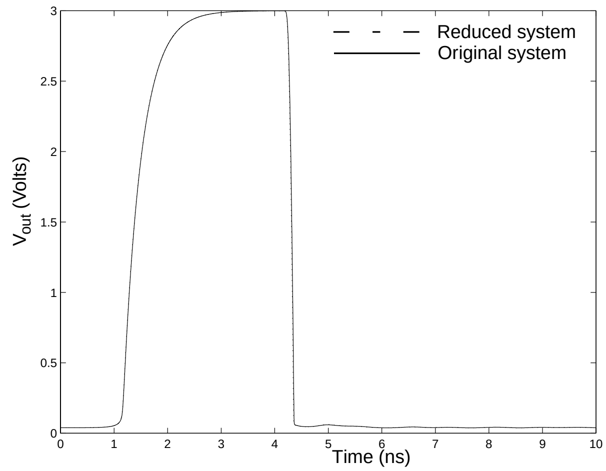

Fig. 7. Transient response at the output node (V_{out})

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